Diagnosing the Intercept Parameters of the Exponential Drop Size Distributions in a Single-Moment Microphysics Scheme and Impact on Supercell Storm Simulations

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Abstract

In this study, power-law relations are developed between the intercept parameter of the exponential particle size distribution and the water content for four hydrometeor categories. The derived relations are implemented within the Milbrandt and Yau microphysics scheme. Simulations of the 3 May 1999 Oklahoma tornadic supercell are performed using the diagnostic relations for rain only, and alternately for all four precipitating species, and results are compared with those from the original single- and double-moment microphysics schemes. Diagnosing the intercept parameter for rain is found to improve the results of the simulation in terms of reproducing the key features of the double-moment simulation while still retaining the computational efficiency of a single-moment scheme. Improvements were seen in general storm structure, cold pool structure and intensity, and the number concentration fields. Diagnosing the intercept parameters for all four species, including those for the ice species, within the single-moment scheme yields even closer alignment with the double-moment simulation results. The decreased cold pool intensity is very similar to that produced by the double-moment simulation, and the areal extent of the storm is more accurately reproduced. This study suggests that significant improvements over traditional fixed-intercept parameters in typical single-moment microphysics schemes can be achieved through the use of diagnostic relations for the parameters of the particle size distribution, without a significant increase in computational cost.
1. Introduction

As numerical weather prediction (NWP) models gain convection-resolving resolutions, the parameterization of microphysical processes becomes critical, especially for precipitation forecasts. Generally, bulk microphysics parameterizations, which specify a particular size distribution for each hydrometeor species and predict certain moments of the size distribution (e.g., the water mass associated with the third moment of the size distribution), are used due to the comparatively high cost of non-bulk spectral or bin models for simulations of three-dimensional moist convection.

The most commonly used particle size distribution (hereafter PSD) for precipitating hydrometeors is the inverse exponential distribution, which can be written as

\[ N(D) = N_0 \exp(-\Lambda D), \quad (1) \]

where \( N_0 \) is the intercept parameter and \( \Lambda \) is the slope parameter of the PSD. Usually the intercept parameter is assigned a fixed value in single-moment microphysics schemes. The most well-known of these exponential distributions is the Marshall-Palmer (Marshall and Palmer 1948) distribution, which specifies \( N_0 \) to be \( 8 \times 10^6 \) drops m\(^{-4}\) for liquid precipitation. However, using this fixed value for the rain intercept parameter has been shown to be restrictive, as the intercept parameter can vary significantly within single precipitation events both spatially and temporally (Tokay and Short 1996; Zhang et al. 2008). Snook and Xue (2008) investigated the effect of varying \( N_0 \) for the rain and hail particle size distributions (PSDs) on storm evolution within high-resolution supercell simulations, focusing on the effect upon tornadogenesis. It was found that in simulations where \( N_0 \) was lowered such that the PSD favored large drops, the resulting cold pools were weaker and the simulations tended to develop into single or multiple supercells, while when \( N_0 \) was raised, the storms transformed to a linear mode during the
simulations. Tornado-like low-level vortices formed in the low-$N_0$ simulations but not in others. Cohen and McCaul (2006) performed simulations using an increased median volume diameter for hail and graupel, which resulted in reduced low-level cooling due to decreased melting. Earlier studies of van den Heever and Cotton (2004) and Gilmore et al. (2004) also found significant sensitivity of simulated supercell storms to PSD parameters. More recently, Van Weverberg et al. (2010; 2011a) also investigated the impact of drop size distributions in single-moment microphysics scheme on precipitation and storm dynamics, and the sensitivity of simulated precipitation systems to microphysics complexity (Van Weverberg et al. 2011b).

The distribution represented by (1) is a subset of the gamma distribution (Ulbrich 1983), which has the form

$$N(D) = N_0 D^\alpha \exp(-\Lambda D), \quad (2)$$

where $\alpha$ is the shape parameter, a dimensionless measure of the spectral width of the distribution. If $\alpha$ is set to 0, the distribution is reduced to the inverse exponential form. The addition of the shape parameter allows the gamma distribution to depict a far greater range of PSDs than the inverse exponential distribution. Mallet and Barthes (2009) applied a maximum likelihood technique to categorize rain drop size distributions (DSDs) from optical disdrometer data, and found that 91% of the measured DSDs were of the gamma type.

Each of the parameters of the gamma distribution varies widely in value in nature, thus parameterizing any of them as constant will introduce a source of error into the microphysics. Ulbrich (1983) calculated a typical range of values for $\alpha$ and $N_0$ of the rain DSD, encompassing relations derived from $Z$-$R$ (reflectivity-rain rate) relationships presented in a number of other studies. Calculated values for $\alpha$ varied from -3.42 to 5.04 (the range was far narrower for studies based on convective rain, with $\alpha$ ranging from 0.40 to 1.63) and values for $N_0$ ranged from $1.29 \times$
10^3 \text{ m}^4 \text{ to } 9.20 \times 10^{12} \text{ m}^4 (7.05 \times 10^6 \text{ m}^4 \text{ to } 2.46 \times 10^8 \text{ m}^4 \text{ for convective rain}). Although some of the variations in parameter values are due to errors in measurements, modeling, and fitting procedures (Cao and Zhang 2009), variation due to physical causes appears to dominate (Zhang et al. 2003 and Milbrandt and Yau 2005a,b).

The role of the shape parameter $\alpha$ was investigated in detail by Milbrandt and Yau (2005a), who showed that the rate of gravitational particle size sorting was dependent on $\alpha$, with the size sorting rate decreasing as $\alpha$ increases and approaching 1 as $\alpha$ becomes large. Size sorting is an intrinsic process within supercell evolution, as witnessed by the occurrence of the differential reflectivity ($Z_{\text{DR}}$) arc within supercells. Kumjian and Ryzhkov (2008) found that the $Z_{\text{DR}}$ arc is associated with large, oblate raindrops and relatively few small drops, suggesting that size-sorting is responsible for the modification of the DSD. Hence it is important that the PSD is parameterized in a realistic manner, allowing all PSD parameters to vary as appropriate.

A way to improve a microphysics scheme is to increase the number of predicted moments of the DSD (Straka and Mansell 2005; Milbrandt and Yau 2005a). How the moments of the PSD are calculated depends on the way the PSD is parameterized. For the gamma distribution the $n^{\text{th}}$ moment is calculated as
\[
M_n = N_0 \Lambda^{-\left(\alpha+n+1\right)} \Gamma\left(\alpha+n+1\right) \tag{3}
\]

Most bulk microphysics schemes, especially those used in operational NWP models, only predict one moment of the distribution, typically the third moment $M_3$, which is proportional to the hydrometeor mixing ratio, $q_x$ (here $x$ denotes one of the species). In this case, it is usually the slope parameter $\Lambda$ that is effectively prognostic while $N_0$ and $\alpha$ are held constant (with $\alpha$ fixed at 0 for the inverse exponential distribution). More recently, microphysical schemes that predict two or more moments have become increasingly popular, particularly for convective scale
modeling. Most of the double-moment schemes available predict both the mixing ratio and number concentration, leaving $\alpha$ held constant while $N_0$ and $\Lambda$ are directly linked to the predicted variables (e.g., Milbrandt and Yau 2005a; Morrison et al. 2005).

Aside from moving to multi-moment schemes, which are computationally expensive, other methods of extending low-moment schemes beyond the fixed single-parameter approach have been attempted. The most common method relates a free parameter in the PSD to another independently predicted PSD parameter. Zhang et al. (2001) investigated relations between the PSD parameters using video disdrometer data collected in Florida and derived a relation between the shape and slope parameters. The $\alpha - \Lambda$ relation was subsequently updated using disdrometer measurements for rain DSDs observed in Oklahoma (Cao et al. 2008).

Zhang et al. (2008) used the same disdrometer data as Cao et al. (2008), gathered in central Oklahoma during the summers of 2005 through 2007, to derive a relationship between the intercept parameter of the inverse exponential PSD and the rain water content (which is proportional to the third moment of the distribution). The diagnostic relation was formed using the method of moment relations, outlined in detail in their paper. Their derived relation for rain was $N_0(M_2, M_4) = 7106 W^{0.648}$, where $N_0$ is measured in # m$^{-3}$ mm$^{-1}$ and $W$ is in g m$^{-3}$. Here $N_0$ is given as a function of $M_2$ and $M_4$ because its relation is derived from these two moments.

In the single-moment version of the Milbrandt and Yau microphysics scheme (Milbrandt and Yau 2005b), described in more detail in section 3, the PSD of each precipitating hydrometeor category is modeled by the gamma distribution with a fixed value of shape parameter $\alpha$. In this paper, we assume the shape parameter $\alpha$ to be zero, leading to the exponential distribution commonly used in single moment schemes.

The $n^{th}$-moment of the exponential DSD is given by
\[ M_n = \int D^n N(D) \, dD = N_0 \Lambda^{-(n+1)} \Gamma(n+1). \quad (4) \]

Given this relation, we can see that the exponential PSD parameters, \( N_0 \) and \( \Lambda \), can be determined using any two moments of the distribution. Given two moments, \( M_j \) and \( M_k \), the PSD parameters can be calculated as

\[ N_0 = \frac{M_j \Lambda^{j+1}}{\Gamma(j+1)}, \quad (5) \]

\[ \Lambda = \left[ \frac{M_j \Gamma(k+1)}{M_k \Gamma(j+1)} \right]^{\frac{1}{k-j}}. \quad (6) \]

The moment estimates from disdrometer measurements contain errors (Zhang et al. 2008), which causes errors in the DSD parameters determined from these estimates. Hence, the middle (second and fourth) moments were used in their study as they contain the least error (Cao and Zhang 2009). The main issue with these observation-based studies is that the diagnostic relations were derived using disdrometer data collected at the surface, primarily for rain. Diagnostic relations derived from three-dimensional data sets for individual species are needed for use within microphysics parameterization schemes.

The goal of this study is to formulate a diagnostic relationship between the intercept parameter and water/ice content for each precipitating hydrometeor species, and implement this within the single-moment Milbrandt and Yau (hereafter MY) microphysics scheme (Milbrandt and Yau 2005b) available within the ARPS nonhydrostatic model (Xue et al. 2000; Xue et al. 2001). It is hypothesized that this should allow a more realistic PSD model than the use of a fixed value of \( N_0 \) for each precipitating hydrometeor species, and will enable a more accurate representation of the particle distributions. To this end, the overall aim of the study is to improve the results of the single-moment microphysics scheme for them to be closer to the results of the
corresponding double-moment microphysics scheme. For the derivation of the PSD parameter relationships in this study, the zeroth and third moments of the inverse exponential PSD are used since these are independently predicted within the double-moment MY microphysics scheme. As the first proof-of-concept attempt and because of the general lack of DSD observations for multiple species in 3D volumes, we use the output of the Milbrandt and Yau double-moment simulations to derive the relations. We use a realistic simulation of a real tornadic supercell storm for the test. The case is described next.

2. The May 3rd 1999 Oklahoma tornadic supercell case

On May 3rd 1999, one of the most significant tornado outbreaks ever to occur in the U.S. caused extensive damage across Oklahoma and Kansas, including the metropolitan areas of Oklahoma City and Wichita. Fifty eight tornadoes struck within the county warning area of the Norman, Oklahoma National Weather Service Forecasting Office over a period of eight hours (Speheger et al. 2002). Sixteen of these tornadoes were rated F2 or greater on the Fujita (1971) scale, including two F4 and one F5 tornadoes. The F5 tornado tracked through the small community of Bridge Creek, parts of Moore, southern Oklahoma City, Del City and Midwest City, causing 36 direct fatalities (Brooks and Doswell 2002) and injuring 583 people. The storm that produced the F5 tornado is the focus of this study.

Observations from the Oklahoma Mesonet (Brock et al. 1995) indicated that the cold pools associated with the supercell which produced the F5 tornado were mainly small and relatively weak. The synoptic setup for the event has been discussed in detail in Thompson and Edwards (2000) and Roebber et al. (2002).

Given the inherent instability present, a gap in the cirrus cloud cover allowed a cumulus tower to develop close to Lawton in Southwestern Oklahoma, around 2030-2045 UTC. This
evolved into the first supercell and after an initial split, rapidly developed into a right-moving supercell – storm A (Thompson and Edwards 2000). Storm A became tornadic and produced at least fourteen distinct documented tornadoes between 2151 UTC on May 3 and 0125 UTC on May 4 (Speheger et al. 2002). The most intense of the tornadoes produced by storm A was A9, the F5 tornado that left a 37-mile trail of destruction through the communities of Bridge Creek, Moore and Oklahoma City. Dawson et al. (2010) simulated a supercell storm within an environment believed to be representative of the environment that storm A developed, and the study found significant sensitivity of the simulated supercell storm to the number of moments predicted with the Milbrandt and Yau microphysics schemes. The predicted cold pool was generally too strong with the single moment scheme while the simulation of the three-moment scheme was found to be the best.

3 Diagnostic relations for intercept parameters

The ARPS implementation of the double-moment MY microphysics scheme has two options, schemes 2a and 2b (hereafter referred to as MY2a and MY2b). Scheme 2a fixes the shape parameter of the gamma distribution, $\alpha$, to a constant value; when $\alpha$ is set to zero this reduces to the inverse exponential distribution. Scheme 2b diagnoses $\alpha$ for each sedimenting hydrometeor species based upon the mean mass diameter $D_{mx}$, in the form $\alpha_x = c_{1x} \tanh \left[ c_{2x} (D_{mx} - c_{3x}) \right] + c_{4x}$, where $D_{mx}$ is in units of mm and $c_{1x}$ through $c_{4x}$ are empirically determined coefficients, detailed in their Table 1 (Milbrandt and Yau 2005a), allowing variability in the PSD shape.

As pointed out earlier, DSD observations for multiple species in 3D volumes are generally unavailable, making it difficult to obtain diagnostic relations for several species that are applicable to the entire storm. As the first proof-of-concept attempt, we use the output of a
double-moment simulation for the May 3, 1999 case to derive the diagnostic relations for use in a single-moment scheme. This also allows us to see how close the results of single-moment scheme with diagnostic relations can be to those of double-moment scheme.

As briefly mentioned earlier, using a single sounding to define the storm environment, Dawson et al. (2010) have shown for the May 3, 1999 case, the simulated supercell is rather sensitive to the use of single, double and triple moment options of the Milbrandt and Yau scheme. Dawson (2009) further performed more realistic simulations of this case using 3D inhomogeneous initial conditions that included the assimilation of radar data. The simulation used three levels of nested grids, at 3 km, 1 km and 250 m horizontal grid spacings, respectively. The North America Regional Reanalysis (NARR) provided the initial analysis background and the boundary conditions for the outermost 3 km grid. The assimilation of Oklahoma City WSR-88D (KTLX) radar data was performed on the 1 km grid, using ARPS Data Analysis System (ADAS) (Brewster 1996) that includes a complex cloud analysis component. The cloud analysis makes use of satellite and radar observations to define cloud and hydrometeor fields and adjust in-cloud temperature and moisture fields (Hu et al. 2006). Conventional observations were analyzed using a successive correction scheme that converges to an optimal interpolation solution (Brewster 1996; Lazarus et al. 2002). These simulations were performed using a full set of physics in the ARPS (Xue et al. 2001). Details of these simulations can be found in Dawson (2009).

In this study, the output of the 250 m simulation using the MY2a scheme (where \( \alpha \) was set to zero) from that study will be used as a “synthetic dataset” to derive the diagnostic relations for the intercept parameters. This simulation will hereafter be referred to as the “reference”
simulation. These relations are then implemented in the single-moment option of the MY scheme for the various experiments in this study.

Model output was taken every 300 seconds throughout the second hour of the reference simulation and collated. For each of the thirteen times, a file containing the zeroth and third moment was produced for each precipitating hydrometeor category, which in the Milbrandt and Yau suite of microphysics schemes includes rain, snow, graupel and hail. Data at grid points in the full domain were included in the file, although since vertical grid stretching was employed, the low levels are more heavily sampled than the upper levels. Points were included provided that a minimum threshold of hydrometeor mixing ratio was met. This threshold was purposely kept low at $1 \times 10^{-5}$ kg kg$^{-1}$ for rain and $5 \times 10^{-5}$ kg kg$^{-1}$ for snow, graupel and hail, in order to accurately represent the full range of mixing ratios produced by the simulation. For each point, the parameters of the inverse exponential distribution were calculated and the water content $W$ was derived using

$$W_x = 1000 \rho_a q_x,$$

where $W_x$ is in g m$^{-3}$, air density $\rho_a$ is in kg m$^{-3}$ and mixing ratio $q_x$ is in kg kg$^{-1}$. As in Zhang et al. (2008), we wish to form a power-law relation between the water content and the intercept parameter of the exponential distribution for each species. In our case, these relations are calculated by performing a least squares minimization on both variables, to give an effective linear relation between the logarithms of the variable pair. Transforming the variables back from logarithmic into linear space provides a power law for $N_{0x}$ in terms of $W_x$. The coefficient and power of the derived $N_{0x}-W_x$ relationships were averaged across the model output times examined in order to give a more general relation for each precipitating hydrometeor category. The plot showing an example of the dependence of $N_{0r}$ on $W_r$ from a single model time exhibits a
high degree of scatter (Fig. 1.) However, the powers and coefficients of the derived relationship did not vary significantly across the range of times examined (not shown), which suggests confidence in the robustness of the derived relation.

The Milbrandt and Yau suite of microphysics schemes contains four frozen hydrometeor categories – ice, snow, graupel and hail – and each of these is handled separately within the ice phase processes. The ice total number concentration, $N_{ii}$, is already diagnosed based upon temperature in the single-moment option, through Cooper’s equation,

$$ N_{ii} = 5 \exp (0.304 (273.15 – \max(233, T))), $$  \hspace{1cm} (8)

where $T$ is the temperature in Kelvin. For rain and the other three frozen categories, the number concentration is calculated as the zeroth moment of the distribution using the intercept parameter of the PSD of that species, which is set to a constant value in the original single-moment microphysics scheme. Recall that with the single-moment option, each of the hydrometeor species is modeled as a gamma distribution with fixed shape parameter of zero and the intercept parameter fixed at a given value for each species (see Table 1). The scatter plots showing the dependence of $N_{0x}$ on $W_x$ for snow, graupel and hail are shown in Figs. 2 through 4. The frozen hydrometeor concentrations are diagnosed based upon the water content within that hydrometeor category. The fixed intercept parameter values used in the original Milbrandt and Yau scheme and the fitted relation are also shown in the figures. Examining Figs. 1-4 and Table 1, it is clear that for each of the hydrometeor species the coefficient differs significantly from the fixed value of $N_{0x}$, by up to three orders of magnitude in the case of graupel.

The derived relation for rain is $N_{0r} = 2.45 \times 10^6 \ W_r^{0.566} \text{ m}^{-4}$, with the power having a standard deviation of 0.153 across the range of times examined. For a sample liquid water content of 1 g m$^{-3}$ the diagnosed $N_{0r}$ would be $2.45 \times 10^6$ m$^{-4}$ compared to the fixed value of 8 \times
10^6 m^{-4}. For N_0, which is usually fixed at 3 \times 10^6 m^{-4}, the derived relation is 3.19 \times 10^9 W_s^{0.755} m^{-4}, which would give N_0 of 3.19 \times 10^9 m^{-4} for a W_s value of 1 g m^{-3}. The standard deviation of the power across the thirteen times is 0.055. The corresponding values for graupel are a usual fixed N_{0g} of 4 \times 10^5 m^{-4}, with the diagnostic relation being N_{0g} = 6.13 \times 10^8 W_g^{0.523} m^{-4} (giving N_{0g} = 6.13 \times 10^8 m^{-4} for W_g = 1 g m^{-3}) and the standard deviation of the power is 0.086. The fixed hail intercept parameter is 4 \times 10^4 m^{-4} and the derived diagnostic relation is N_{0h} = 5.13 \times 10^6 W_h^{0.467} m^{-4} (such that N_{0h} = 5.13 \times 10^6 for W_h = 1 g m^{-3}), with a standard deviation of the power of 0.056.

4. Numerical experiments

In order to test the diagnostic N_0 relations, several simulations of the May 3rd 1999 case were performed. An important aim of this study is to implement the diagnostic relations for N_0 within a single-moment microphysics scheme and determine how well such a single-moment scheme reproduces the results of double-moment scheme. Two baseline simulations are first produced, using the original MY1, MY2a schemes, respectively. The results of the diagnostic N_0 simulations are compared against these to gauge the impact and effectiveness of the diagnostic relations. Another simulation was performed using the original MY1 scheme with N_{0r} artificially lowered from 8 \times 10^6 m^{-4} to 4 \times 10^5 m^{-4}. Two further simulations were performed using the derived diagnostic N_{0x} relations, one in which only the warm rain processes used the diagnostic relation, and one in which the intercept parameters for both warm rain and ice processes, including 4 precipitating hydrometeors total, are diagnosed. Details of the simulations and the nomenclature used can be found in Table 2.

Each simulation was performed using the same model (ARPS) as the 250-m reference simulation that produced the synthetic dataset used for fitting the diagnostic relations. However,
rather than the complex full physics real-data setup of the reference simulation, the simulations herein used the same single sounding and thermal bubble initialization procedure as in the idealized experiments of Dawson et al. (2010), and were performed at the same horizontal resolution of 500 m. The use of this idealized model setup allows more control over the experiments, and represents at least part of the natural variability among storms and their environment. It also facilitates direct comparison with the results of Dawson et al. (2010). The simulations MY1_orig and MY2_orig are identical to MY1 and MY2 from Dawson et al. (2010) and are repeated here mainly for convenience.

Fifty three vertical levels were used with vertical grid stretching employed, giving a vertical resolution of 20 m at the surface that decreases to 800 m at the upper boundary, the same as in the 250 m reference simulation. The fourth-order monotonic computational mixing scheme of Xue (2000) was utilized with a coefficient of 0.0015 s\(^{-1}\). Each simulation used a different microphysical scheme setup, although all of these were based on either the MY1 or MY2a schemes (Milbrandt and Yau 2005b). The initial ellipsoidal thermal bubble used to trigger the storm had a maximum potential temperature perturbation of 4 K, horizontal radius of 10 km and vertical radius of 1.5 km, and was centered 1.5 km above the surface, 35 km from the west edge and 25 km from the south edge of the domain, which was of horizontal dimensions 128 km × 175 km. The sounding used in the simulation was extracted from a 1-hour forecast of an earlier 3 km real-data simulation of this case, at a location upstream of the low-level inflow of the storms. This was the same sounding used in Dawson et al. (2010), and full details of the original real-data simulation can be found in Dawson et al. (2007).
5. Results and discussion

a. Reflectivity structure

The MY1_orig simulation produced a storm that did not show realistic mature supercell characteristics, as can be seen by the reflectivity structure (Fig. 5a). It must be noted that the storm in MY1_orig was beginning to decay by one hour into the simulation, due to the strong surging cold pool cutting off the updraft at the low levels in this simulation. The area of maximum surface reflectivity, located around 60 km from the western and 80 km from the southern boundary of the domain, shows a maximum around 55 dBZ, although the lateral extent of the forward flank reflectivity region is smaller than we would expect for an event of this type.

The forward flank reflectivity region is more realistically represented in MY2_orig (Fig. 5b) than it is in the single-moment simulation, although in MY2_orig the forward flank reflectivity region is larger than expected. Morrison and Milbrandt (2011) found that the MY schemes produce long and narrow forward flank regions. This does not affect the key point of this paper since we are primarily interested in how well the results of the single-moment scheme can match the double-moment, as a proof-of-concept. The relative improvement in the forward flank reflectivity region in MY2_orig could be because MY1_orig decayed shortly after the time we are examining here, while that in MY2_orig the storm correctly sustained. However, there are many processes that cannot, due to the nature of single-moment microphysics, be well represented within the single-moment scheme, such as size sorting. In contrast, the double-moment scheme does parameterize size sorting, although when the shape parameter $\alpha$ is set to 0 (as in MY2_orig), excessive size sorting tends to occur (Milbrandt and Yau 2005a). The effect of this excessive size sorting can be witnessed in Fig. 5b, in the sharp gradient at the leading edge of the forward flank reflectivity. Simulations of this case performed using the triple-moment
Milbrandt and Yau microphysics scheme produced a more gradual and realistic reflectivity gradient in this region (see Fig. 3 in Dawson et al., 2010), corresponding more closely to radar observations of this case (Dawson et al. 2010).

MY1_N0, in which the rain intercept parameter value is reduced, is seen to produce a more realistic storm structure than MY1_orig, particularly on the leading edge or forward flank of the storm. The maximum reflectivity in this simulation is 61 dBZ, which is close to that of MY2_orig, and significantly increased from that of MY1_orig. Since the only difference between the two simulations is the reduction in the (fixed) value of $N_0$, this would seem to indicate that the default value of $8 \times 10^6$ m$^{-4}$ for $N_0$ is too high, at least for the type of severe convective case being simulated here. As was shown by Snook and Xue (2006), altering the intercept parameters for rain and hail PSDs can have a large impact on tornadogenesis, so we expect that reducing $N_0$ will result in significant changes in several fields. Despite the improved overall structure of the storm, the areal extent of the forward flank reflectivity region in MY1_N0 is still significantly smaller compared to that in MY2_orig.

The reflectivity structure produced by diag_rain is broadly similar to that of MY1_N0, although the lateral extent of the storm is slightly increased. The maximum reflectivity is similar to that of both MY1_N0 and MY2_orig. The forward flank region is better represented in diag_rain than in the fixed-$N_0$ experiments, with the strength of the east-west reflectivity gradient in the forward flank region being decreased in diag_rain (the reflectivity decreases to zero more gradually from west to east), which corresponds more closely to base reflectivity observations from KTLX (0.5° tilt, not shown).

While diagnosing the intercept parameter of the rain PSD produces improvement over the original fixed-$N_0$ single-moment simulation, the improvement is mostly limited to the lowest
levels, since the same ice phase processes are used in \textit{diag\_rain} as \textit{MY1\_orig}, and little improvement is observed above the freezing level. At the surface, improvements in \textit{diag\_rain} in the forward flank region are still relatively small; it is hypothesized that this is because much of the liquid water in this region originates from melting and fallout from the anvil region, which is not well simulated by the fixed-$N_0$ experiments.

To investigate possible further improvements to the solution in the forward flank region, the diagnostic relations for the intercept parameters of snow, hail and graupel are implemented in \textit{diag\_all}. Examining the reflectivity structure generated by \textit{diag\_all}, (Fig. 5e) it is clear that the inclusion of diagnostic intercept parameters for graupel, snow and hail has a large effect on the lateral extent of the storm and general storm structure. The overall reflectivity structure is seen to resemble that of \textit{MY2\_orig} more closely than any other simulations. Particular improvement (again, “improvement” is defined relative to the \textit{MY2\_orig} simulation) is noticed in the forward flank region of the storm, with that of \textit{diag\_all} much larger in size than in the other single-moment simulations. This would suggest that by diagnosing the intercept parameters for the frozen categories, we are able to more accurately represent the ice processes that contribute to fall-out from the anvil. However, the maximum reflectivity seen in \textit{diag\_all} is reduced by around 10 dBZ from that of \textit{MY2\_orig}, to 43 dBZ, which is lower than we would expect for a storm of this nature. The lower reflectivity in \textit{diag\_all} than \textit{diag\_rain} is a consequence of a significant decrease in rain water mixing ratio seen in this simulation (Fig. 6). The domain-maximum rain water mixing ratio at the surface in \textit{diag\_all} is 0.5 g kg$^{-1}$, over four times less than the maximum of 2.2 g kg$^{-1}$ in \textit{diag\_rain}. The domain-maximum rain water mixing ratio in \textit{diag\_all} is less than that in single-moment simulations at the 2 km height (Fig. 6) and it continues to decrease toward the surface. The \textit{MY2\_orig} simulation shows a similar domain-maximum rain water mixing ratio
to \textit{diag\_all}, however, in this simulation it increases toward the surface. This reduced rain water mixing ratio directly impacts the reflectivity, causing the reduced reflectivity seen in \textit{diag\_all}.

Although it is expected that the greatest improvement will be seen at levels below the melting layer, the reflectivity structure is also examined at 5.5 km height. \textit{MY1\_N0} and \textit{diag\_rain} show little improvement over \textit{MY1\_orig} (Figs. 7a, 7c, 7d), which is expected as only the rain PSD is altered in these simulations. As at the surface, the forward flank reflectivity region is larger in extent in \textit{MY2\_orig} than the single-moment simulations (Fig. 7b), although the inclusion of the diagnostic relations for the frozen hydrometeors in \textit{diag\_all} (Fig. 7e) shows an increase in the areal extent of this region. We note here that the reflectivity calculation in each case is consistent with the DSD models used in each simulation.

\textit{b. Cold pool structure}

The cold pool is discussed in terms of the equivalent potential temperature perturbation ($\theta_e'$) fields at the surface, as in Dawson et al (2010), since the $\theta_e'$ field also takes into account moisture effects. Here, $\theta_e$ is calculated according to Bolton (1980). According to Dawson et al. (2010), James and Markowski (2010) and McCaul and Cohen (2002), since $\theta_e$ is a conserved variable even in the presence of moist processes, $\theta_e$ can be used as an estimation for the source region of the air in the cold pool; below the level of minimum $\theta_e$, lower values of $\theta_e$ indicate a higher source level of the surface air. Of course, the above statement assumes that $\theta_e$ is conserved in the downdraft, which is not strictly true. If turbulent mixing with the environment occurs, or any process that causes a departure from pseudoadiabatic descent (such as the melting of frozen hydrometeors) then $\theta_e$ will not be exactly conserved.

The cold pool produced in \textit{MY1\_orig} is seen to be very strong (Fig.8a). In fact, it is the strength of the cold pool which cuts off the updraft after one hour and causes the simulated storm
to decay. This process is believed to be responsible for the mis-alignment of the low-level gust frontal forcing and the mid-level mesocyclone that causes the failure in tornadogenesis in Snook and Xue (2008). \textit{MY1\textsubscript{orig}} also produces a strong cold pool in the forward flank region of the storm, which was not observationally verified for this case. Mobile mesonet observations from May 3\textsuperscript{rd} indicate that the cold pools in the forward flank region of the storm for this event were small and weak, with maximum temperature deficits of no more than 2-3 K (Markowski 2002). The temperature deficits in \textit{MY1\textsubscript{orig}} are close to 8 K (not shown).

The cold pool in \textit{MY2\textsubscript{orig}} is very weak in comparison, with a maximum temperature deficit of 0.7 K (not shown), which is weaker than the cold pool observed by the mobile mesonet (Markowski 2002). The cold pool in the 250 m reference simulation is stronger than that seen in \textit{MY2\textsubscript{orig}}, and corresponds more closely to mobile mesonet observed temperature perturbations (Dawson 2009) which suggests that \textit{MY2\textsubscript{orig}} simulation at 3600s is producing a storm that is low-precipitation (LP) in character, as storm A was during its developing stage. The reduced strength of the cold pool in the forward flank region in \textit{MY2\textsubscript{orig}} as compared to \textit{MY1\textsubscript{orig}} also corresponds more closely to the mobile-mesonet-observed weak cold pool in this region (Markowski 2002). The $\theta_e$ deficit is reduced to 18 K for \textit{MY2\textsubscript{orig}} from 30 K for \textit{MY1\textsubscript{orig}}.

The cold pool produced by \textit{MY1\_N0} is weaker than that of the original single-moment scheme, with a maximum temperature deficit of 4 K. Snook and Xue (2008) found that simulations performed using reduced values for $N_{0r}$ and $N_{0h}$ tend to produce a relatively weak cold pool when compared to simulations performed using increased intercept parameter values. They concluded that this is due to distributions that favor fewer large drops (as characterized by a reduced intercept parameter) possessing a reduced total hydrometeor surface area when compared to the same mass of water distributed as a large number of smaller drops. This reduced
surface area limits the potential for evaporative cooling, which is one of the most important mechanisms for cold pool formation. Ice processes also play an important role in cold pool formation, and this is discussed next.

The cold pool structure of diag_rain is intermediate between MY1_orig and MY1_N0. The maximum temperature deficit is 7.1 K (not shown), which is slightly reduced from that of MY1_orig but higher than that of MY1_N0. When examining the cold pool in terms of $\theta_e'$, the cold pool produced by diag_rain is again larger than that produced by MY1_N0, with a higher maximum intensity (Figs. 8c, 8d). It seems that diagnosing $N_0$ does not improve the simulation in terms of the cold pool structure and intensity as much as simply reducing $N_0$ by a factor of 20; diagnostic treatment of the frozen hydrometeors may still be needed for more realistic simulation of the cold pool. While the cold pool in MY1_N0 is weaker, we know the fixed $N_0$ value throughout the domain is not physical.

The cold pool produced by diag_all that does include diagnostic treatment of ice PSDs is much weaker than the cold pool witnessed in any of the previous single-moment simulations. The maximum temperature deficit is 1.3 K (not shown), which is close to the maximum temperature deficit of 0.7 K in MY2_orig, and is closer to the observed cold pool intensity (Markowski 2002). Since the single-moment microphysics schemes, with their default intercept parameter values at least, tend to produce unrealistically strong cold pools, the fact that diagnostic relations of the intercept parameters implemented within a single-moment scheme produce much more realistic cold pools shows the promise of the diagnostic approach. The $\theta_e'$ field (Fig. 8e) for diag_all also shows a very similar cold pool structure to that of MY2_orig, with the maximum $\theta_e$ deficit of -18.9 K (Fig. 8e) matching closely the maximum $\theta_e$ deficit of -18 K produced by MY2_orig. Although reducing the strength of the cold pool may not be desirable
in all cases, the goal of this study is to recreate the key features of the double-moment scheme, which in this case presented with a cold pool of significantly reduced strength from the single-moment simulation.

c. Rain water number concentration

While the total number concentration of rain water ($N_{tr}$) is independently predicted within $MY2_{\text{orig}}$, it is not independently predicted in the single-moment scheme. $N_{tr}$ can be diagnosed from the mixing ratio and intercept parameter for each species of single-moment simulations using

$$N_{tr} = N_{0x}^{3/4} \left( \frac{\rho_x q_x}{\pi \rho_s} \right)^{1/4}. \quad (9)$$

The maximum number concentration of rain water one hour into the simulation is 4167 m$^{-3}$, in $MY1_{\text{orig}}$, which is an order of magnitude higher than that produced by double-moment simulation $MY2_{\text{orig}}$ (Figs. 10a and 10b). This is consistent with the fact that the double-moment simulation produced a storm that was low-precipitation (LP) in character, as was found in Dawson et al. (2010), rather than the traditional ‘classic’ supercell (Lemon and Doswell 1979).

Being directly proportional to $N_{0x}$ (see Eq. 9), $N_{tr}$ is an order of magnitude smaller than that of $MY1_{\text{orig}}$ in $MY1_{N0}$; the maximum number concentration is a little over double that of $MY2_{\text{orig}}$ at 483 m$^{-3}$ (Fig. 9c). The striking differences in storm structure and cold pool between $MY1_{\text{orig}}$ and $MY1_{N0}$ exemplify the large impact that PSD parameters can have upon the simulation results. Altering $N_{0x}$ affects the simulation both directly through the impact on $N_{tx}$, and also indirectly through feedback from processes such as evaporative effects.
The maximum $N_r$ produced by $\text{diag\_rain}$ was greater than that of $MY1\_N0$, at 3085 m$^3$, and that produced by $\text{diag\_all}$ was reduced from those of $MY1\_orig$ and $\text{diag\_rain}$, although at 937.9 m$^3$ it is still higher than the double-moment simulation. As the contribution from the rain category is handled the same way in $\text{diag\_rain}$ and $\text{diag\_all}$, this result indicates that the treatment of the frozen hydrometeor categories has a large impact on rain water concentration through fallout and melting.

We note here that even though similar results, in terms of the cold pool strength and precipitation structure, seem obtainable by choosing a smaller value of the rain intercept parameter, in practice, it is difficult to predetermine what value should be used and the constant fixed values are not necessarily realistic. For this reason, we believe the diagnostic relations hold more promise; to say the least they provide spatial variability to the PSD parameters that appear to be more physical.

In terms of each of the variables examined in this study, consistent improvements are seen in $\text{diag\_rain}$ over $MY1\_orig$. Further improvements in examined variables in $\text{diag\_all}$, except for the weak reflectivity core, bring the results of that simulation further in line with results produced by the double-moment simulation, consistent with the goal of this study. This is encouraging, and it is suggested that the method of diagnosing the intercept parameters based on independently predicted variables merits further investigation.

**d. Evolution of surface RMS errors**

As the goal of this study was to recreate the key features of the double-moment simulation through use of diagnostic relations for $N_{0r}$ within the single-moment framework, the evolution of RMS errors in the surface reflectivity, temperature and $\theta_e$ fields is examined. As we seek to mimic the output of $MY2\_orig$, RMS errors are calculated on the difference between the
surface-level reflectivity, temperature and $\theta_e$ fields of $MY2_{orig}$ and the four single-moment simulations.

In terms of reflectivity, although $diag\_all$ has the highest RMS error during storm development, by 60 minutes into the simulation, $MY1\_N0$, $diag\_rain$ and $diag\_all$ all show similar RMS errors, each noticeably outperforming $MY1\_orig$ (Fig. 10). The reflectivity fields from $MY1\_N0$ and $diag\_rain$ were very similar (Figs. 5c, 5d), so their closely correlated RMS errors are unsurprising. It is thought that while the maximum reflectivity in $diag\_all$ is significantly reduced at the low levels from that of all the other simulations, the increased areal extent of the reflectivity is more closely aligned with that of $MY2\_orig$, which goes some way to reduce the impact of the decreased maximum reflectivity value on the RMS error.

The RMS errors in the temperature and $\theta_e$ fields indicate that throughout the simulation period, the cold pool in $diag\_all$ most closely matched that of $MY2\_orig$ (Fig. 11), while the cold pool in $MY1\_N0$ resembled the double-moment simulation more closely than that of $diag\_rain$. All of the simulations outperformed $MY1\_orig$, with the RMS error in temperature of $MY1\_orig$ consistently double that of $MY1\_N0$ and 40% higher than $diag\_rain$ (Fig. 11). These results lend weight to the earlier assertion that diagnosing the intercept parameter for rain alone does not offer enough improvement in the cold pool over selecting a fixed, reduced (but arbitrary and perhaps tunable after the event) value for $N_{0r}$, and the form of the frozen hydrometeor distributions must also be considered in order to produce a more realistic cold pool intensity.

e. Relative computational costs

The main motivation for designing a diagnostic single-moment scheme is computational efficiency. The goal is to achieve performance similar to a double-moment scheme at the cost of a single-moment scheme. Table 3 lists the timing statistics for three simulations using the
Milbrandt-Yau single-moment (MY1) and double-moment (MY2) schemes, and the standard Lin et al (1983) ice microphysics scheme available in the ARPS. The times for the scalar advection, subgrid-scale turbulence and computational mixing terms (sadv+mix), for the microphysics (mphysics) and for all other parts of the model are given. It can be seen that the time spent on advecting and diffusing the scalar prognostic variables is increased from 2788 s in MY1 to 4283s in MY2, a 53% increase. This is because of the increase in the number of prognostic variables by 6, for the total number concentrations of cloud water, cloud ice, rainwater, snow, graupel and hail in MY2. It is surprising that MY1 spent more time (8335s) on the microphysical processes than MY2 (5827s). An examination of the codes reveal that the MY1 scheme was implemented more for convenience than for efficiency. It borrows codes from the MY2 scheme as much as possible, and expresses the microphysical terms in terms of the mixing ratios and total number of concentrations, and when needed deriving the latter from the fixed intercept parameters. As a result, there are more computations in MY1 scheme than in MY2 scheme in the microphysics package. Obviously, for efficiently implemented single-moment schemes, the single-moment option should be no more expensive than the double-moment counterpart. For this reason, we ran the standard Lin scheme in the ARPS for reference. The Lin scheme in the ARPS is highly optimized, and the time spent on the scheme is only 1625s, about 28% that of MY2 and 20% that of MY1. For this reason, we formed a new column called MY1_Lin in Table 3, which is the same as that for MY1 except that the microphysics time is an average of those of MY1 and Lin schemes, assuming that the microphysics portion of MY1 can be optimized to be between the MY2 and Lin schemes. With the numbers of MY1_Lin, the overall cost using MY2 scheme will be about 50% more than using the optimized MY1 scheme (the second from last column in Table 3). If the MY1 scheme can be further optimized to reach the efficiency of the ARPS Lin scheme,
then the total cost of using MY1 will only be about $1/1.87 = 0.53$ times the cost of MY2 scheme. Because the additional cost of diagnosing the intercept parameters is minimal, by using our diagnostic intercept parameter scheme, one can potentially achieve a factor of 1.5 to 2 speedup overall, compared to the corresponding double-moment scheme.

6. Summary and conclusions

The overall goal of this study was to establish and utilize a relationship between the PSD parameters and the hydrometeor mass variables typically predicted in single-moment microphysics schemes in the hope of producing results closer to those of double-moment schemes. The PSD parameter chosen was the intercept parameter of the size distribution. It has already been shown from disdrometer measurements that there is a measurable positive correlation between the two variables (Zhang et al. 2008) and in this study we derived such relationships based on the output of a realistic two-moment simulation. The original single-moment Milbrandt and Yau microphysics scheme uses a fixed value for the intercept parameter of the distribution of each hydrometeor species. Two control simulations were run using the original single and double-moment microphysics schemes and these were used as the basis against which all subsequent simulations in this study are compared.

Simply reducing the fixed value of $N_{0r}$ by a factor of 20 showed improvements over $MY1_{orig}$ in terms of producing results more closely aligned with those of the double-moment simulation (more realistic reflectivity structure, reduced cold pool strength, reduced number concentration.)

Diagnosing the intercept parameter of the rain PSD produced improvements upon the original fixed-$N_{0r}$ single-moment simulation, however, these are limited to the lowest levels. The addition of a diagnostic relation for $N_{0x}$ for the frozen hydrometeor categories is seen to have a
greater impact on the simulation results than diagnosing $N_{0r}$ only. Improvements are seen at low levels as well as above the melting layer, due to improvements in the structure of the frozen categories bringing improvements in the rain category through fallout and melting.

The use of diagnostic relations for several frozen species was seen to significantly increase the lateral extent of the storm in diag_all over the other single-moment simulations. Extending the diagnostic intercept parameter relation to the frozen hydrometeors also has a large positive impact on the cold pool structure. From Snook and Xue (2008, 2006), it is known that altering the intercept parameter of any of the hydrometeor distributions can have a large effect on the cold pool, as altering the shape of the distribution directly affects the rate of evaporative cooling.

While diag_rain produced a cold pool smaller in size and strength than that of MY1_orig, diagnosing the frozen category intercept parameters further reduced both the intensity and size of the cold pool, bringing the temperature deficit in line with that of MY2_orig. The cold pool in MY2_orig was seen to be very weak in the forward flank region, which agrees with surface mesonet observations from the event (Markowski 2002), hence improvements in the representation of the forward flank region through diagnosing the intercept parameters of the frozen species is encouraging.

While improvements are seen for the use of the diagnostic relations within these simulations, questions still exist about the representativeness of the derived $N_{0r} - W_s$ relations, especially in light of the significantly decreased reflectivity core in diag_all. This could be investigated through the performance of simulations for a number of other cases. If the relations derived here are too case-specific, the inclusion of other cases into the study would allow us to derive a more general relation for each species. If relations are derived that diagnose $N_{0r}$ for each
of the precipitating hydrometeor species and are applicable to a range of different cases (for example severe convective storms, squall line cases, stratiform precipitation and winter storms), then this method shows considerable promise in improving the results of a single-moment microphysics scheme, with computational cost increases only one-third of that of moving to a double-moment scheme. It is hoped that through further exploration of the parameter space a set of relations can be derived that will be applicable to a range of situations. A further simulation was performed to investigate the specificity of the derived relations. The same model setup was used, with \( N_{0x} \) for each species diagnosed using the derived relations, but with the power in the diagnostic relations (see Table 1) set to 1 for each species. Results (not shown) were very similar to that of diag_all, which indicates that results seem to be not very sensitive to the exact value in the coefficient of the power law but further investigation is needed to ascertain this.

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Fig. 1. Dependence of rain intercept parameter ($N_0r$) on water content ($W$). Points are directly fitted using the zeroth and third moments of the exponential DSD to obtain the $N_0r-W$ relation. The dashed line is the derived fitted relation (in linear space) and the bold line shows the fixed value of $N_0r$ used in the original Milbrandt and Yau single-moment scheme.

Fig. 2. As in Fig. 1, but for snow.

Fig. 3. As in Fig. 1, but for graupel.

Fig. 4. As in Fig. 1, but for hail.

Fig. 5. Reflectivity and horizontal wind vectors at the surface (plotted every 2.5 km) at 3600 s into the five simulations: a) $MY1_{\text{orig}}$, b) $MY2_{\text{orig}}$, c) $MY1_{\text{N0}}$, d) $\text{diag}_{\text{rain}}$ and e) $\text{diag}_{\text{all}}$.

Fig. 6. Rain mixing ratio profiles in the lowest 2 km.

Fig. 7. Reflectivity and horizontal wind vectors at 5.5 km height (plotted every 2.5 km) for the five simulations as in Fig. 5.

Fig. 8. Surface equivalent potential temperature perturbation (shaded) and reflectivity (contours, 10 dBZ increment) for the five simulations as in Fig. 5.

Fig. 9. Rain water number concentration (contour interval reflected in panels) at the surface for the five simulations shown in Fig. 5. Domain-maximum rain water concentrations are shown in the lower right of each panel.

Fig. 10. Evolution of the RMS error in reflectivity for each of the four single-moment simulations when compared to $MY2_{\text{orig}}$.

Fig. 11. Evolution of the RMS error in equivalent potential temperature for each of the four single-moment simulations when compared to $MY2_{\text{orig}}$. 
Table 1. The fixed $N_{0x}$ values used in the original Milbrandt and Yau single-moment microphysics scheme, and the derived diagnostic relation for $N_{0x}$ (based on $W_x$) for rain, snow, graupel and hail.

<table>
<thead>
<tr>
<th>Species</th>
<th>Default fixed $N_{0x}$ values</th>
<th>Diagnostic relation derived from real data simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>$N_{0r} = 8 \times 10^6 \text{ m}^{-4}$</td>
<td>$N_{0r} = 2.45 \times 10^6 W_r^{0.566} \text{ m}^{-4}$</td>
</tr>
<tr>
<td>Snow</td>
<td>$N_{0s} = 3 \times 10^6 \text{ m}^{-4}$</td>
<td>$N_{0s} = 3.19 \times 10^9 W_s^{0.755} \text{ m}^{-4}$</td>
</tr>
<tr>
<td>Graupel</td>
<td>$N_{0g} = 4 \times 10^5 \text{ m}^{-4}$</td>
<td>$N_{0g} = 6.13 \times 10^8 W_g^{0.523} \text{ m}^{-4}$</td>
</tr>
<tr>
<td>Hail</td>
<td>$N_{0h} = 4 \times 10^4 \text{ m}^{-4}$</td>
<td>$N_{0h} = 5.13 \times 10^6 W_h^{0.467} \text{ m}^{-4}$</td>
</tr>
</tbody>
</table>

Table 2. Description of the microphysical setup used in the five simulations that were performed at 500m horizontal resolution.

<table>
<thead>
<tr>
<th>Experiment Name</th>
<th>Details of microphysics scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MY1_orig$</td>
<td>Original single-moment Milbrandt and Yau scheme.</td>
</tr>
<tr>
<td>$MY2_orig$</td>
<td>Original double-moment Milbrandt and Yau scheme.</td>
</tr>
<tr>
<td>$MY1_N0$</td>
<td>Single-moment with reduced fixed value of $N_{0r} = 4 \times 10^5 \text{ m}^{-4}$.</td>
</tr>
<tr>
<td>$diag_rain$</td>
<td>Single-moment with intercept parameter diagnosed for rain only.</td>
</tr>
<tr>
<td>$diag_all$</td>
<td>Single-moment with intercept parameter diagnosed for all precipitating hydrometeor categories.</td>
</tr>
</tbody>
</table>

Table 3. Timing statistics for three simulations using Lin, Milbrandt-Yau single-moment (MY1) and Milbrandt-Yau two-moment (MY2) schemes for the scalar advection, subgrid-scale turbulence and computation (sadv+mix), microphysics (mphysics) and all other parts of the model. Another column, MY1 Lin, is included which is the same as MY1 except that the mphysics time is an average of MY1 and Lin scheme times. The last two columns show the time ratios for the three portions and the total between MY2 and MY1 Lin, and MY2 and Lin, respectively. The timings of the diagnostic MY1 scheme are very similar to that of MY1 and are therefore not shown. The times shown are in second and the times were collected by running the ARPS model using a single processor.

<table>
<thead>
<tr>
<th>Timing</th>
<th>Lin (time/percent)</th>
<th>MY1 (time/percent)</th>
<th>MY1_Lin (time/percent)</th>
<th>MY2 (time/percent)</th>
<th>MY2/MY1_Lin ratios</th>
<th>MY2/Lin ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>sadv+mix</td>
<td>3121/50.4%</td>
<td>2788/22.6%</td>
<td>2788/36.0%</td>
<td>4283/37%</td>
<td>1.53</td>
<td>1.37</td>
</tr>
<tr>
<td>mphysics</td>
<td>1625/26.2%</td>
<td>8335/67.4%</td>
<td>3726/48.0%</td>
<td>5827/50%</td>
<td>1.56</td>
<td>3.59</td>
</tr>
<tr>
<td>Other</td>
<td>1450/23.4%</td>
<td>1239/10.0%</td>
<td>1239/16.0%</td>
<td>1490/13%</td>
<td>1.20</td>
<td>1.02</td>
</tr>
<tr>
<td>Total</td>
<td>6195/100%</td>
<td>12363/100%</td>
<td>7753/100%</td>
<td>11601/100%</td>
<td>1.50</td>
<td>1.87</td>
</tr>
</tbody>
</table>
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