Assimilating Polarimetric Radar Data with an Ensemble Kalman Filter: OSSEs with a Tornadic Supercell Storm Simulated with a Two-Moment Microphysics Scheme

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Abstract

The impact of assimilating additional differential reflectivity $Z_{DR}$ data from a polarimetric radar on the analysis of a tornadic supercell storm using an ensemble Kalman filter (EnKF) is studied in an observing system simulation experiment (OSSE) framework assuming a perfect forecast model. A double-moment microphysics scheme is used to allow for proper simulation of polarimetric signatures. Root-mean square errors of analyzed state variables are calculated and the structure and intensity of analyzed fields and derived quantities are examined. Compared to the baseline experiment assimilating radial velocity and reflectivity only, the assimilation of additional $Z_{DR}$ further reduces the errors of all state variables. The analyzed hydrometer fields are improved in both pattern and intensity distributions. Polarimetric signatures including $Z_{DR}$ and $K_{DP}$ columns, and $Z_{DR}$ arc in the supercell are much better reproduced.

Sensitivity experiments are performed that exclude the updating of hydrometeor number concentrations by $Z_{DR}$ or of state variables not directly linked to $Z_{DR}$ via observation operators. The results show that if number concentrations are not updated together with the mixing ratios, most of the benefit of assimilating $Z_{DR}$ is lost. Among other state variables, the updating of water vapor mixing ratio $q_v$ has the largest positive impact while the impact of updating vertical wind $w$ comes in second. The updating of horizontal wind components or temperature has much smaller but still noticeable impact. Reliable flow-dependent cross-covariances among the state variables and observation prior as derived from the forecast ensemble and used in EnKF are clearly very beneficial.

Key points:
• ZDR assimilation improves the analyses for nearly all state variables of a simulated supercell storm.
• The updating of water value mixing ratio \( q_v \) has the largest impact followed by the updating of vertical velocity \( w \).
• The hydrometeor total number concentrations should be updated together with the mixing ratios within EnKF.

1. Introduction
The forecast accuracy of high-resolution numerical weather prediction (NWP) models highly depends on the model initial state, especially for short-lived convective storms; the accuracy of initial microphysics (MP) state variables is key to successful short-range forecasting of precipitating systems (Sun et al. 2013). Radar is the only observational platform that can capture the internal structures of convective storms, at high spatial and temporal resolutions (Stensrud et al. 2009; Hu et al. 2006). Many studies have shown that radar data assimilation (DA) greatly reduces the spin up time of model and improves short-range precipitation forecasts (Dixon et al. 2009; Xiao et al. 2008; Hu and Xue 2007; Zhu et al. 2015; Xue et al. 2003; Hu et al. 2006).

To obtain additional information on precipitation MP, the entire U.S. operational WSR-88D Doppler radar network has been upgraded to dual polarization a few years ago (ROC 2013). In Europe, the number of operational dual-Doppler radars has grown steadily (Huuskonen et al. 2014). More countries such as China are in the process of upgrading its operational radars to dual polarization (Zhao et al. 2019). Compared to single-polarization radar, a dual-polarization radar measures hydrometeor particle scattering at both horizontal and vertical polarizations, and can thereby provide information on the shape and other characteristics of hydrometeor particles. From polarimetric radar data (PRD), rain drop/particle size distributions (PSDs) and related properties can be better retrieved (Huang et al. 2019; Zhang et al. 2019; Cao et al. 2013), as can be hydrometeor classification within storms (Ryzhkov and Zrnic 1998; Vivekanandan et al. 1999).

It is expected that the assimilation of PRD into NWP models would help improve the analysis (initialization) and prediction of precipitating systems. So far, studies on the assimilation of PRD are relatively few, however, and most of the studies assimilate PRD indirectly, i.e., retrieval of model state variables from data is performed first before assimilation. Wu et al (2000) assimilated rain and ice mixing ratios retrieved from \( Z_H \) and differential reflectivity \( Z_{DR} \), assuming that only two hydrometeor categories, i.e., rain and ice, existed. In their study, the positive impact of assimilating PRD did not, however, last long in the forecast, and error associated with the very simple ice MP scheme used was suggested to be a reason.

Li and Mecikalski (2010) assimilated \( Z_H \) and \( Z_{DR} \) data based on warm-rain-only observation operators implemented within the Weather Research and Forecasting (WRF) three-dimensional variational (3DVAR) DA system. With the assimilation of both \( Z_H \) and \( Z_{DR} \), in-storm structures were said to be better analyzed and short-range precipitation forecast was also improved. More recently, Li et al (2017) developed an observation operator for specific differential phase (\( K_{DP} \)) that includes an ice phase (snow) and found positive impact of assimilating extra \( K_{DP} \) data using WRF 3DVAR on analyzed rainwater in the lower troposphere and snow in the mid- to upper troposphere for a mesoscale convective system. The impact was limited to the examination of analysis increments of rainwater and snow at a single time though and the impact was limited to a portion of the analyzed storm due to limited data coverage\(^1\).

\(^1\) We note that their reflectivity operators including liquid and ice contained a significant error, so their results should be viewed with caution.
Some studies have attempted to assimilate information derived from polarimetric signatures within convective storms. For example, in intense supercell storms, a column of high $Z_{DR}$ or $Z_{DR}$ column is often found in the region of intense updraft, corresponding to large raindrops that can be lofted above the freezing level in the form of supercooled liquid water (Kumjian and Ryzhkov 2008). In a proof-of-concept study, Carlin et al. (2017), the moisture and temperature adjustments within the ARPS cloud analysis system (Hu et al. 2006) were modified to be based on the detection of $Z_{DR}$ columns for two tornadic supercell storm cases. Both analyses and forecasts of the storms were improved compared to the use of the original cloud analysis in both cases. While the procedure appears to be effective for tornadic supercell storms, it will be hard to apply, however, to weaker precipitating systems where $Z_{DR}$ column is much less pronounced or absent. Such methods also rely on empirical relations between PRD and model state variables.

More direct and quantitative use of PRD is desirable through direct assimilation. Direct DA methods compare simulated observations from the model state variables against observations, and make adjustments to the state variables to achieve optimal fit of the analyzed state to observations and the prior guess of the state subjecting to the weights related to their respective errors (Kalnay 2002). Forward observation operators are needed to simulate PRD from the model state variables, and the forecast model should have a reasonable capability in simulating observed polarimetric signatures. Jung et al. (2008a) developed PRD observation operators based on calculations of electromagnetic wave scattering by hydrometeors then used power law functions to fit backscattering amplitudes to obtain computationally more efficient operators. The contributions of wet snow and wet grapuel/hail are also included. In Jung et al. (2010b), more accurate observation operators based on rigorous scattering calculations using the T-matrix method (Vivekanandan et al. 1991) are developed. Details of observation operators will be given in section 2a. Jung et al. (2010b) compared the performances of single-moment (SM) and double-moment (DM) microphysics in terms of the simulated polarimetric signatures. Their results showed that certain polarimetric signatures such as $Z_{DR}$ arc, $\rho_{hv}$ (cross-correlation coefficient) rings can only be correctly simulated by DM scheme. Simulated PRD can have large uncertainties and can vary significantly with the use of MP scheme, however (Putnam et al. 2017b; Putnam et al. 2017a).

The observation operators for PRD variables such as $Z_{DR}$ are highly nonlinear. To variationally assimilate PRD, linear tangent and adjoint of the observation operators are needed, and the high nonlinearity often causes convergence problems with the variational minimization (Liu et al. 2019). With the ensemble ensemble Kalman filter (EnKF) method that has been shown to work well with complex MP schemes (Tong and Xue 2005), linear tangent or adjoint of the observation operators is not needed. EnKF also has the ability to directly update state variables not directly involved in the observation operators, through ensemble-estimated flow-dependent background error covariances, even in the presence of complex mixed-phase microphysics (Tong and Xue 2005). Jung et al. (2008b) first assimilate PRD using EnKF with a SM MP schemes with positive impacts achieved and Jung et al. (2010a) demonstrated the benefit of PRD in improving the estimation of both microphysical state variables and PSD parameters associated with a SM MP scheme. Both of these studies assimilated simulated PRD.

Certain polarimetric signatures that depend on hydrometeors size sorting (Dawson et al. 2014), such as the $Z_{DR}$ arc in the supercell storms, can only be properly simulated using multi-moment MP schemes (Jung et al. 2010b; Putnam et al. 2014). In the only published study that directly assimilates real polarimetric observations using EnKF, Putnam et al. (2019) showed that the analyzed $Z_{DR}$ structures including the $Z_{DR}$ arc in a supercell storm are improved with additional $Z_{DR}$ assimilation. The study also showed that the analyzed rain mean mass diameter is higher in
the $Z_{DR}$ arc region and the total rain number concentration is lower downshear in the forward flank, agreeing with observational estimations. Biases do exist in their EnKF analyses that require further investigations (Putnam et al. 2019), however.

As far as we know, Putnam et al (2019) is the only formally published study that examines the impact of directly assimilating additional PRD using EnKF combined with a multi-moment MP scheme. Many issues, including analysis biases, remain that require further studies as they pointed out. Being a real-data-based study, detailed verification of analyzed state variables, especially those of MP, is difficult, because of the lack of truth. Errors in the observational data can complicate the issues. To better understand the behaviors and impacts of assimilating additional PRD, observation system simulation experiments (OSSEs) can be very helpful. While Jung et al. (2008b) and Jung et al. (2010b) examined the impacts of PRD data via OSSEs, their EnKF DA studies had limitations with the use of a SM MP scheme. For the above reasons, OSSEs are performed in this study with EnKF combined with a DM MP scheme and compatible observation operators, examining the impact of directly assimilating additional $Z_{DR}$ data. Additional sensitivity experiments are performed to see the impacts of updating total number concentrations (the additional PSD moment associated with a DM MP schemes) and updating state variable not directly linked to PRD via observation operators.

The rest of this paper is organized as follows. In section 2, the observation operators used in this study together with configurations of the OSSE experiments are described. The results of control and sensitivity experiments examining the impacts of PRD assimilation are presented and discussed in section 3. Summary and conclusions are given in section 4.

2. Experiment configuration and settings

a) The truth simulation and observation operators

For the OSSEs, a truth simulation is produced using the Advanced Regional Prediction System (ARPS, Xue et al. 2003) initialized from a sounding for the 1977 Del City, Oklahoma supercell storm (Ray et al. 1981), as given in Xue et al (2001). A 4-K ellipsoidal thermal bubble with radii of 10 km in the horizontal directions and 1.5 km in the vertical direction is used to initiate the storm. Most of the configurations are inherited from Tong and Xue (2005) except for the MP scheme used and the grid configuration. The SM Lin MP scheme is replaced by DM Milbrandt-Yau MP scheme (Milbrandt and Yau 2005); as mentioned earlier, DM schemes can much better reproduce $Z_{DR}$ signatures (Jung et al. 2010b). The simulation domain has 105x103x53 grid points and the horizontal grid spacing is 1 km. The average vertical grid spacing is 400 m, and is stretched from 50 m at the surface.

For DM schemes, the shape parameter of three-parameter gamma distributions assumed of most hydrometeor PSDs is typically assumed constant (with zero being assumed most often). In this study, the shape parameter for rainwater in the Milbrandt-Yau DM scheme is set to 2 while for other hydrometers it is set to zero. Studies have found that most DM schemes tend to overestimate simulated reflectivity (eg: Putnam et al. 2017b; Brown et al. 2016). One of the reasons, according to Brown et al (2016), is that most schemes tend to produce a higher frequency of large raindrops than observed. Setting the rain shape parameter to 2 helps reduce the number of large raindrops and in turn reflectivity.

The observation operator for radial velocity is the same as that used in Jung et al (2008a). However, there are some differences from OSSE experiments of Jung et al (2008a) where the observation operators for radar reflectivity are calculated using a fitted approximation to T-matrix scattering amplitudes for rain and Rayleigh approximation for ice hydrometeors. This
approximation may result in some error (Putnam et al. 2019). In this study, more advanced
observation operators using look up tables calculated from T-matrix method are used (Jung et al.
2010b). In the following are the formula for radar reflectivity factors at the horizontal and vertical
polarizations based on the full T-matrix algorithm:

\[ Z_{H,s} = \frac{4\lambda^4}{\pi^3} |K_w|^2 \int_0^{D_{\text{max}}^s} [A |f_{a,s}(\pi)|^2 + B |f_{b,s}(\pi)|^2 + 2C \text{Re}[f_{a,s}(\pi)f_{b,s}^*(\pi)]]n(D)dD, \] (1)

\[ Z_{V,s} = \frac{4\lambda^4}{\pi^3} |K_w|^2 \int_0^{D_{\text{max}}^s} [B |f_{a,s}(\pi)|^2 + A |f_{b,s}(\pi)|^2 + 2C \text{Re}[f_{a,s}(\pi)f_{b,s}^*(\pi)]]n(D)dD, \] (2)

where

\[ A = \frac{1}{8} (3 + 4 \cos 2\phi \cos 2\sigma - \cos 4\phi \cos 8\sigma), \]

\[ B = \frac{1}{8} (3 - 4 \cos 2\phi \cos 2\sigma + \cos 4\phi \cos 8\sigma), \]

\[ C = \frac{1}{8} (1 - \cos 4\phi \cos 8\sigma). \]

Here, \( \lambda \) is the wavelength of the radar and we assume a 10.7 cm wavelength S-band radar. \( K_w \)
=0.93 is the dielectric factor for water. \( \phi \) is the mean canting angle and \( \sigma \) is the standard deviation
of the canting angle. \( \phi = 0 \) is assumed for all species. \( \sigma \) are 0°, 20°, 60°, and 60° for rain, snow,
graupel and hail, respectively. \( |\ldots| \) represents the modulus of complex number while \( \text{Re}[\ldots] \)
represents the real part. Superscript * implies the conjugate. Subscript \( s \) can be rain (r), rain-snow mixture (rs), dry snow (ds), rain-graupel mixture (rg), dry
graupel (dg), rain-hail mixture (rh) and dry hail (dh). \( D \) is the diameter of a given hydrometeor and
\( D_{\text{max}} \) is the maximum diameter of each hydrometeor category. In this paper, the maximum
diameters of rain drops, snow aggregates, graupels and hailstones are assumed to be 8, 30, 50 and
70 mm, respectively. \( n(D) \) is the number concentration of the hydrometeor at diameter \( D \). To
numerically integrate Eqs. (1) and (2), the integral ranges are partitioned into 100 bins. The
backscattering amplitudes of each species with assumed drop size for polarizations along the major
\( f_o(\pi) \) and minor \( f_h(\pi) \) axes are precomputed at the center of each size bin and stored in look-up
tables. For melting species including rain-snow, rain-graupel and rain-hail mixtures, the same
tables are constructed at the uniform 5% water fraction interval. The fraction of water of each ice
specie is calculated as \( f_{w,s} = \frac{q_r}{q_r + q_{iw}} \). Here, \( q_r \) is the mixing ratio of rain while \( q_{iw} \) is one of the
ice hydrometers. More details on the PRD observation operators can be found in Jung et al
(2010b).

Once the radar reflectivity factors of all hydrometeor categories are calculated. The
dBZ at horizontal and vertical are computed as follows:

\[ Z_H = 10 \log_{10} (Z_{h,r} + Z_{h,rs} + Z_{h,ds} + Z_{h,rg} + Z_{h,rg} + Z_{h,rh} + Z_{h,dh}) \] (3)

\[ Z_V = 10 \log_{10} (Z_{v,r} + Z_{v,rs} + Z_{v,ds} + Z_{v,rg} + Z_{v,rg} + Z_{v,rh} + Z_{v,dh}) \] (4)

The differential reflectivity \( Z_{DR} \) is calculated according to the following formula:

\[ Z_{DR} = Z_H - Z_V. \] (5)
b) EnKF experiment settings and DA experiments

In this study, we use the ARPS EnKF package (Tong and Xue 2005; Xue et al. 2006) which uses the ensemble square root filter algorithm (Whitaker and Hamill 2002). The EnKF experiments employ 40 members in this study. With the DM Milbrandt-Yau MP scheme, the analysis variables include the three-dimensional wind components ($u$, $v$, and $w$), pressure ($p$), potential temperature ($\theta$), water vapor mixing ratio ($q_v$), as well as microphysical state variables including mixing ratios of cloud water ($q_i$), rainwater ($q_r$), cloud ice ($q_s$), snow aggregate ($q_n$), graupel ($q_g$) and hail ($q_h$), and their total number concentrations ($N_{q_c}$, $N_{q_r}$, $N_{q_i}$, $N_{q_s}$, $N_{q_n}$, $N_{q_g}$, and $N_{q_h}$, respectively). Spin-up ensemble forecasts are run for 20 minutes, starting from initial ensemble states defined by the sounding profiles plus smoothed Gaussian random perturbations added in regions where observed reflectivity is larger than 10 dBZ. The mean standard deviations of added $u$, $v$, and $w$ perturbations are 2 m s$^{-1}$ and that of $\theta$ is 2 K. For water vapor and hydrometer mixing ratios, the mean standard deviations of added perturbations are 0.0006 kg kg$^{-1}$. Consider the large uncertainty of number concentrations, we did not add perturbation to those variables. After the 20-min long spin-up forecasts, EnKF DA cycles are run over a 90-min period assimilating radar data every 5 minutes, corresponding to the model storm time period of 20 through 110 minutes. Similar settings were in our earlier OSSE studies (Jung et al. 2008a; Tong and Xue 2005). The 90-min assimilation period is chosen mostly based on the life cycle of the storm. In the truth simulation, the main storm reaches its mature stage between 60 to 100 min. After that, the storm begins to weaken and move out of the simulation domain.

PRD from an assumed S-band radar with its center located in the southwest corner ($x=2$, $y=2$) are simulated from the truth simulation output, using the observation operators described in section 2a. Eleven elevations are assumed, based on the WSR-88D radar VCP-11 scan mode. Radar observation errors are assumed to be 1 m s$^{-1}$, 3 dBZ, 0.2 dB for radial velocity $V_r$, and $Z_H$ and $Z_{DR}$ in terms of standard deviation, respectively, and random errors of the corresponding magnitudes are added to the simulated PRD observations and assumed in the EnKF experiments. $V_r$ observations are assimilated where observed $Z_H > 10$ dBZ. For $Z_{DR}$, only values larger than 0.3 dB are used because smaller values tend to be very noisy. The covariance localization radii for radar observations are set to 4 km in the horizontal and 2 km in the vertical direction using the correlation function of Gaspari and Cohn (Gaspari and Cohn 1999) for all state variables. The 4 km horizontal grid spacing spans 4 grid interval in this study, which is consistent with most past studies in terms of grid intervals. For example, in Tong and Xue (2005) and Jung et al. (2008a), 6 to 8 km were suggested when a 2 km horizontal grid spacing was used. Sobash and Stensrud (2013) suggest suggested 12 to 18 km horizontal radii when using a 3 km grid spacing. We have tested larger and smaller horizontal localization radii. The state analysis errors were found to be significantly larger when using a 6 km radius while the results using 3 km were slightly worse. To help maintain ensemble spread, multiplicative inflation (Anderson 2001; Tong and Xue 2005) is applied to all model state variables except for number concentrations, using an inflation coefficient of 1.2.

Table 1 lists all experiments presented in this paper. Experiment VrZh assimilates $V_r$ and $Z_H$ data while experiment VrZhZdr assimilates additional $Z_{DR}$ data. Both experiments update a full set of state variables in the model. Experiment VrZhZdr is considered a control experiment while VrZh is a reference for comparison purpose. Additional sensitivity experiments are conducted to help better understand how the assimilation of $Z_{DR}$ improves the analysis. The first sensitivity experiment VrZhZdr_NoNt, excludes the updating of total number concentrations of the hydrometeors $N_q$, by $Z_{DR}$ observations compared to experiment VrZhZdr. $N_q$ are still updated by
$V_r$ and $Z_H$ though, just not by $Z_{DR}$. $N_q$, arise from the use of a DM scheme and adds additional complexity to the DA problem. The number concentrations of hydrometers have very wide dynamic ranges, varying from 0 to as large as $10^{12}$ m$^{-3}$, implying that the relations between them and PRD observations can be very nonlinear. Updating both mixing ratios and total number concentrations at the same time may or may not be beneficial, especially when the correlations are unreliable or inconsistent with each other. $V_r$Z$dr$No$N_t$ serves to examine the benefit, if any, of updating the total number concentrations using $Z_{DR}$ observations.

Other sensitivity experiments serve to examine the impact of updating other state variables using $Z_{DR}$. Experiments VrZhZdr_NoW, VrZhZdr_NoUV, VrZhZdr_NoQv and VrZhZdr_NoPt excludes the updating of vertical velocity $w$, horizontal wind components $u$ and $v$, water vapor mixing ratio $q_v$ and potential temperature $\theta$, respectively. In an intense tornadic supercell, a $Z_{DR}$ column typically exists in the updraft region (Kumjian and Ryzhkov 2008), indicating strong positive correlation between upward motion and $Z_{DR}$. Updraft regions are also associated with high moisture values. The largest theoretical benefit of EnKF method compared to 3DVar and some of the other methods lies with the use of ensemble-derived correlations among all state variables, and hence among observation priors and state variables, which allows for the updating of state variables not directly observed (or involved in the observation operators). For such updating to be beneficial, the ensemble-derived correlations have to be sufficiently accurate and reliable. This second group of sensitivity experiments are designed to test the impacts of updating state variables that are not directly linked to $Z_{DR}$ observations via the observation operators.

3. Results of EnKF analyses

a) Evaluation of $Z_{DR}$ assimilation in the control experiment

Figure 1 shows the ensemble mean analysis and forecast RMSEs of model state variables during the assimilation cycles. Following Tong and Xue (2005) and many other studies, the RMSEs are calculated over grid points where the true reflectivity is greater than 10 dBZ, which roughly covers the precipitation regions. For most variables, VrZhZdr (red lines), which assimilates additional differential reflectivity, produces consistently better analyses and forecasts than VrZh (black lines), especially in later cycles. Such results are quite similar to those of Jung et al. (2008b) which examined the impact of assimilating additional $Z_{DR}$ data in OSSEs employing a SM MP scheme, except that RMSEs of most variables in the first few cycles are also reduced here. In Jung et al (2008b), the assimilation of $Z_{DR}$ does not show positive impact until later cycles. Additionally, we also examine the RMSEs of total number concentrations of hydrometer variables, which were not predicted in Jung et al (2008b). Here, for most number concentrations, the $Z_{DR}$ assimilation shows neutral to positive impact. Among them, the number concentration for graupel, $N_q$, is improved most. As we will discuss later, it is probably benefiting from better analyses of liquid hydrometeor species, which in turn lead to more accurate analyses of ice hydrometeor species. In Fig. 2, we show the RMSEs of the analyses and forecasts throughout the DA cycles in terms of radar observed variables, i.e., the verifications in observation space. The results are consistent with the results in terms of the state variables, as shown in Fig. 1; the assimilation of $Z_{DR}$ data further reduces the differences between the analyses and forecasts from the observations in the observation space.

Fig. 3 shows the vertical profiles of ensemble mean analysis and forecast RMSEs at 80 minutes, again averaged over grid points with observed $Z_H$ exceeds 10 dBZ. At this time, the RMSEs of most variables have stabilized (Fig. 1). It can be seen that the errors at most levels for most variables are reduced from the additional $Z_{DR}$ assimilation. The largest improvements are mostly
located where the errors are largest. As Jung et al. (2008b) pointed out, the direct improvements from ZDR assimilation are mainly to those highly correlated variables such as $q_v$ and $q_r$ at the lower levels, where the ZDR signatures are most prominent (given that large ZDR is mostly associated with large raindrops). With more accurate analyses at the lower levels, the analysis fields at upper levels can also be improved through the dynamic interactions in the forecast model. The weak and unreliable correlations between ZDR and ice fields are the upper levels during the earlier cycles might be the reason for larger errors in $q_v$ before 45 min (Fig. 1k) while the errors become smaller in later cycles. Note that in Figs. 1-3, the results of VrZhZdr_NoNt are also included which will be discussed in section 4b later.

In Fig. 4 and Fig. 5, we further examine the impact of ZDR assimilation on the polarimetric signatures of simulated storm. At 80 minutes (Fig. 4a), the ZDR arc is not clearly seen in the truth simulation. We can see a narrow high ZDR band along the edge of 35 dBZ reflectivity. Between this ZDR band and main storm, there is a weak ZDR area (green to light yellow) which is due to hail falling and melting in this region. At 110 minutes (Fig. 4b), high ZDR (red color) extends all the way from the forward flank reflectivity core to the southern edge of forward flank precipitation region; in fact, it extends beyond the 35 dBZ reflectivity contour, suggesting the existence of a relatively small number of large rain drops there, giving rise to relatively high ZDR values. Along this edge, an arc of high ZDR is often observed, due to hail stone and rain drop size sorting (Dawson et al. 2014). For both analysis times, experiment VrZhZdr with additional ZDR assimilation shows better ZDR structure than that of VrZh, especially for later analysis time. For experiment VrZh, the pattern of high ZDR area (red color) is not as good as experiment VrZhZdr when compared to truth simulation at 110 minutes.

The ZDR structure near hook echo region is similar to the classic supercell storm structure for both truth simulation and EnKF analyses (Fig. 4d, e, f). Here, we only display small hook area at 80 minutes because it shows a clear ZDR column (see Fig. 6d). At 110 minutes, the ZDR columns are not obvious (not shown). High ZDR values are located at the leading edge of the high ZH hook (black contours) (Fig. 4d). Experiment VrZh shows generally similar patterns but the intensity is clearly underestimated for both ZH and ZDR (Fig. 4b, e). With additional ZDR assimilation, the shape of ZDR arc in experiment VrZhZdr looks closer to that of truth than in experiment VrZh (Fig. 4c). The intensity of ZDR in the hook echo region is also much enhanced in VrZhZdr (Fig. 4f). Moreover, the ZH pattern is also been improved. The 35 dBZ ZH contours in the southeast edge are much closer to those of truth (Fig. 4a, b, c), and the ZH intensity in the hook region is greatly enhanced (Fig. 4 d, e, f). In all, ZDR assimilation improves the polarimetric signatures of the simulated storm, especially in the hook echo and forward flank regions.

The vertical cross sections of analyzed ZH, ZDR and specific differential phase, $K_{DP}$, in the hook echo region through the low-level ZH and ZDR maximum centers at 80 minutes are shown in Fig. 6. In general, both experiments VrZh and VrZhZdr produce similar patterns of these fields. However, the intensities of ZDR and $K_{DP}$ are clearly underestimated in VrZh. Here $K_{DP}$ is not directly assimilated, but derived from analyzed model state variables using the same equation as in Jung et al. (2010b). For ZH, the maximum values above 60 dBZ are right below the 0 °C contours in all cases (Fig. 6a, b, c). The 45 dBZ ZH contours (orange) extend up to above the -20 °C in the truth (see Fig. 6a) and in VrZhZdr (Fig. 4c), but only to -10 °C line in VrZh (see Fig. 6b), indicating the analyzed storm is less intense in VrZh. The improved vertical structure of ZH indicates better analysis of the hydrometeor fields, which we will show more in Fig. 7. The assimilation of ZDR data also results in a more intense core updraft that is closer to the truth as indicated by the 10 m s\(^{-1}\) $w$ contours in Fig. 6a-c. With a stronger updraft, particles are more likely transported to high
altitudes and also likely undergo more growth before falling to the ground. Associated with the
updraft is a column of high \( Z_{DR} \) values that extend to the -10 °C level in the truth (Fig. 6d) and in
\( VrZhZdr \) (Fig. 6f), while that in \( VrZh \) is clearly weaker (Fig. 6e). Also, a column of high \( K_{DP} \) is
also better reproduced in \( VrZhZdr \) (Fig. 6i) than in \( VrZh \) (Fig. 6h) compared to the truth (Fig. 6g).
High \( K_{DP} \) is mostly associated with high liquid water content, which is linked to intense updraft
and heavy precipitation.

Fig. 7 shows the analyzed cloud water, hail and rain water mixing ratios from \( VrZh \) and
\( VrZhZdr \) in the same vertical cross sections as Fig. 6, as compared to the truth. Since only \( Z_{DR} \)
observations larger than 0.3 dB are assimilated, the direct impact from \( Z_{DR} \) are mostly limited in
the lower levels (c.f., Fig. 6d). However, its benefit could be spread to the higher levels through
spatial and cross-variable correlations, and through dynamic interactions within the forecast model.
Fig. 7 shows that the cloud ice field is better analyzed all the way to the cloud top at ~ 9 km height
in \( VrZhZdr \) (Fig. 7c) and while that in \( VrZh \) is mostly limited to below – 20 °C level or about 6.5
km height; its maximum value is also too low (Fig. 7b). For hail, \( VrZhZdr \) also much better
reproduces the vertical distribution and intensity (Fig. 7f) than \( VrZh \) (Fig. 7e); the latter severely
underestimates hail at the higher levels. For rainwater, the analysis of \( VrZhZdr \) is also better,
although the differences are smaller (Fig. 7i, h). Overall, the assimilation of additional \( Z_{DR} \)
produces analyses of the supercell storm whose intensity and structure are much closer to the truth,
in terms of both observed parameters (\( Z_H \) and \( Z_{DR} \)) and model state variables.

b) The updating of hydrometeor number concentrations with \( Z_{DR} \)

For the DM MP scheme, the hydrometeor number concentrations are part of the forecast
variables which increase the degrees of freedom of the model state. As pointed out earlier, the
values of number concentrations show a great range of variability. Additionally, for DM schemes,
\( Z_{DR} \) depends mostly on the slope parameter of PSD which is a strong function of the third moment,
the mass mixing ratio (Jung et al. 2008b). It is not certain whether the updating of number
concentrations by EnKF will improve the overall analysis. The RMSEs for most state variables
and also for radar observed variables of the experiment \( VrZhZdr\_NoNt \) that excludes the updating
of number concentrations are shown in Fig. 1 to Fig. 3. It can be seen that without updating \( Nq_r \),
the RMSE curves of \( VrZhZdr\_NoNt \) (blue lines) are more close to those of \( VrZh \) than \( VrZhZdr \)
during the later DA cycles for most variables (Fig. 1). Similar is true in terms of radar observed
variables \( Vr, Z_H \) and \( Z_{DR} \) (Fig. 2). For \( w, q_r \) and \( q_h \), the RMSEs of \( VrZhZdr\_NoNt \) even exceed
those of \( VrZh \) in some of the cycles (Figs. 1c, 1i, 1l). The deterioration of the analyses in
\( VrZhZdr\_NoNt \) are more clear in the vertical profiles of RMSEs at 80 minutes (Fig. 3). For \( w \) and
most ice state variables, the RMSEs in \( VrZhZdr\_NoNt \) are larger than those of \( VrZh \) at the upper
levels (Fig. 3) while for \( q_r \) this happens at the mid-levels (Fig. 3i). These results suggest that
updating both mixing ratios and total number concentrations of hydrometeors species associated
with a DM MP scheme together when assimilating \( Z_{DR} \) is important; when only mixing ratios are
updated, most of the benefit of assimilating \( Z_{DR} \) data is lost, and for some variables, that analyses
may be even worse than not assimilating \( Z_{DR} \) data at all. This is presumably because serious
imbalance or inconsistency is created between mixing ratios and corresponding number
concentrations when only the former are updated.

Fig. 8 shows the analyzed rainwater number concentrations \( Nq_r \) at 3 km height, and hail
number concentrations \( Nq_h \) at \( z = 6 \) km from \( VrZhZdr \) and \( VrZhZdr\_NoNt \), as compared to the
truth. For the truth, highest \( Nq_r \) values are found in the southwest part of the supercell storm near
the hook echo region and in the northwest part, corresponding to heavy rainfall in the rear flank
and forward flank downdraft regions, respectively (Fig. 8a). The patterns of analyzed $N_{q_r}$ are similar (Fig. 8b, c) although there is a larger area of over-estimation in the forward flank region while the high values in the rear flank region are under-estimated in VrZhZdr_NoNt (Fig. 8c). Both over-estimation and under-estimation are much less in VrZhZdr (Fig. 8b). The hail number concentration $N_{qh}$ for the truth exhibits moderately high values in the southeastward-spreading forward flank and storm anvil regions at the 6 km height (Fig. 8d) while in the hook echo region, a ring of high $N_{qh}$ is found around a $N_{qh}$ hole, while the highest values found on the west and southwest sides of the hole (Fig. 8d). The hole should be associated with bounded weak echo region typically found in intense supercell storms where hydrometers are mostly absent being swept away by the intense updraft. Within VrZhZdr_NoNt, the ‘ring’ structure is over-estimated (Fig. 8f) although the pattern of $N_{qh}$ in the forward flank region is a little better in VrZhZdr_NoNt (Fig. 8f) than in VrZhZdr (Fig. 8e). Overall, $N_{q_r}$ and $N_{qh}$ are better analyzed in experiment VrZhZdr than in VrZhZdr_NoNt.

Fig. 9 explain the possible reasons. Here, we calculate the correlation coefficients between the $Z_{DR}$ observation prior and the hydrometeor state variables $\rho (Z_{DR}, N_{q_r})$ from the forecast ensemble in a vertical cross section passing through $Z_{DR}$ prior which is located in the $Z_{DR}$ column at (x, y, z) = (34, 22, 3.5) km. In general, $Z_{DR}$ has clearly higher correlations to $N_{q_r}$, $N_q$, and $N_{qh}$ than to $N_{q_l}$, $N_h$, and $N_{qh}$. This is because $Z_{DR}$ is most sensitive to raindrop sizes and high $Z_{DR}$ is found where there are a large number of large rain drops. Many large drops originate from the melting of falling hail stones (Dawson et al. 2014). A column of high correlation is found for $\rho (Z_{DR}, N_{q_r})$, $\rho (Z_{DR}, N_{q_l})$ and $\rho (Z_{DR}, N_{qh})$ near the main updraft. For $N_{q_l}$, $N_{q_k}$ and $N_{q_h}$, the correlations are weaker, and non-zero values are mostly found above the freezing level (Fig. 9). The coherent structures in the correlations between $Z_{DR}$ and $q_r$, and between $Z_{DR}$ and $N_{q_r}$ suggest that the flow-dependent error covariances estimated and utilized within the EnKF should be physically reasonable, and hence the updating of $N_{q_r}$ in addition to $q_r$ can be beneficial.

We also examine correlations between $Z_{DR}$ at a 1.8 km height and hydrometeor state variables in a vertical cross section in the forward flank high $Z_{DR}$ region (Fig. 10). The cloud water at this point is zero for all members. Therefore, the correlation is zero and is not shown. For other hydrometeor variables, similar to the point in the hook echo region, correlations $\rho (Z_{DR}, N_{q_r})$ and $\rho (Z_{DR}, N_{qh})$ are clearly higher than $\rho (Z_{DR}, N_{q_l})$ and $\rho (Z_{DR}, N_{q_h})$. The patterns of correlation $\rho (Z_{DR}, q_r)$ are also very similar to the corresponding $\rho (Z_{DR}, N_{q_r})$ except that those of hail show opposite signs of correlation near the surface (Fig. 10e, j). The negative correlation between $Z_{DR}$ at 1.8 km and $q_l$ at the lower levels is consistent with the fact that hail stones tend to contribute little to $Z_{DR}$ due to tumbling (which is the cause of $Z_{DR}$ hole within supercell storms as a significant hail signature (Kumjian and Ryzhkov 2008)), while the positive correlation between $Z_{DR}$ and $N_{qh}$ suggests that when a larger number of small hailstones exist, melting hailstones will cause less reduction to $Z_{DR}$. Given that large correlations between $Z_{DR}$ and mixing ratio and between $Z_{DR}$ and number concentration for rainwater and hail simultaneously, updating mixing ratios without updating corresponding number concentrations will create imbalances between different moments of the hydrometeor PSDs which in turn will negatively affect the analysis and forecast states.

c) The updating of other state variables with $Z_{DR}$ assimilation

Fig. 11 shows that analysis and forecast RMSEs of sensitivity experiments without updating certain state variables when assimilating $Z_{DR}$. RMSEs for experiment VrZhZdr are shown in black lines while those for VrZhZdr_NoPt, VrZhZdr_NoUV and VrZhZdr_NoW are shown in color. Among all potential temperature, water vapor, vertical and horizontal wind components, the
the updating of water vapor $q_v$ has the greatest impact. The RMSEs from VrZhZdr_NoQv (solid purple) are significantly larger for almost all forecast times and state variables and the differences are larger in later cycles. The updating of $w$ has the second largest impact as the RMSEs of VrZhZdr_NoW (red) are noticeably larger for most variables especially during the intermediate cycles. The updating of horizontal wind components and potential temperature has less impact as the RMSEs of VrZhZdr_NoPt (solid green) and VrZhZdr_NoUV (blue) are rather close to those of VrZhZdr. These results are reasonable since water vapor is the primary fuel for intense convection while $w$ provides the best measure of the intensity of convection. Given that VrZhZdr produces overall the lowest RMSEs, all state variables should be updated when assimilating $Z_{DR}$, at least when no model error is present and the ensemble-estimated covariances are reasonably accurate.

The vertical RMSE profiles up to 5 km height at 80 minutes are shown in Fig. 12. Here, we focus on the low levels where $Z_{DR}$ has largest impacts. Consistent with Fig. 11, experiment VrZhZdr_NoQv has the largest errors at essentially all vertical levels shown. Experiment VrZhZdr_NoW produces second largest RMSEs for most variables at most levels. The updating of potential temperature $\theta$ has the third largest impact (e.g., on $q_v$ in Fig. 12d, on $p'$ in Fig. 12f, and $q_h$ in Fig. 12l), although for some variables not updating $\theta$ made little differences (e.g., for $q_c$ in Fig. 12g and $q_f$ in Fig. 12h). The updating of $u$ and $v$ has limited impact from lower to middle levels. In experiment VrZhZdr_NoUV, the analysis RMSEs are close to those of VrZhZdr below 2 km, but larger above 2 km for variables including $q_v$ (Fig. 12d), $q_c$ (Fig. 12g) and $q_f$ (Fig. 12h). This is better illustrated in Fig. 13 which show the correlations between the $Z_{DR}$ and wind components, the mixing ratio $q_v$ and potential temperature perturbation $\theta'$. The $Z_{DR}$ point is the same as the point in Fig. 9. For $q_v$ and $w$, they show high and continuous correlation regions from the bottom to the top. For $u$, $v$ and $\theta$, the high correlation areas are clearly reduced and mostly located in the lower levels.

In Fig. 14, we further examine the impact of not updating certain variables on the dynamic structures of analyzed storm. Here, the vertical vorticity $\zeta$ at 2 km height in the main updraft region is shown, indicating low-level mesocyclone structure and intensity. Also plotted are the vertical velocity $w$ and horizontal winds. The truth shows an ellipse shaped structure of $\zeta$ with its center located to south of $w$ maximum (Fig. 14a). With all state variables updated in EnKF, experiment VrZhZdr obtains very similar structures of $\zeta$ and $w$ with the horizontal winds flowing around the north side of the updraft core (Fig. 14b). Without updating horizontal winds in VrZhZdr_NoUV when assimilating $Z_{DR}$ data, the overall structures of $\zeta$ and $w$ and horizontal winds are not too different from those of VrZhZdr except that their intensities are somewhat underestimated (Fig. 14c). The impact of not updating $\theta$ in VrZhZdr_NoPt by $Z_{DR}$ is similar to not updating $u$ and $v$ (impact is relatively small), although the maximum $w$ is slightly overestimated according to the $w$ maximum values shown in the plots (Fig. 14f). Compared to $u$, $v$ and $\theta$, the impact of not updating $w$ or $q_v$ when assimilating $Z_{DR}$ is much larger. Without updating $w$, the $\zeta$ pattern appears more circular and the updraft is more concentrated than maximum is overestimated (Fig. 14d). Without updating $q_v$, the shapes of $\zeta$ structure and updraft region are still close to those of VrZhZdr and truth, but the maximum $\zeta$ is most overestimated among the sensitivity experiments, and $w$ maximum is also overestimated (Fig. 14e), although slightly less so than in VrZhZdr_NoW.

The above results indicate the analyzed flow structures and intensity in the main updraft region are directly linked to the updating of $w$ and $q_v$. This is more obvious in vertical cross sections. Fig. 15 shows the vertical cross sections of $\zeta$ and $w$ fields through maximum of three dimensional $w$ in the $y$ direction. In both truth and experiment VrZhZdr, a $\zeta$ maximum is found at $\sim$1.6 km level
which corresponds to relatively strong vertical motion there, and the fields in VrZhZdr match those of truth very closely. Except for experiment VrZhZdr_NoW, the general patterns of $\zeta$ and $w$ in other sensitivity experiments are similar except for under-estimation of the low-level vorticity strength, especially in VrZhZdr_NoQv and VrZhZdr_NoPt. In VrZhZdr_NoW, mid-level ($z \sim 5$ km) $w$ is over-estimated by nearly 50%, as is the column of high vertical vorticity (Fig. 15f). Fig. 14 and Fig. 15 provide more concrete ideas on the large impact of updating or not updating $w$ and $q_v$ by $Z_{DR}$, results that are consistent with earlier findings based on RMSEs. This further confirms that there are reliable, strong ensemble-derived correlation between $w$ and $Z_{DR}$ that enables improved analysis of $w$ and other fields by $Z_{DR}$ observations.

Based on the above results, all model state variables should be updated when assimilating $Z_{DR}$ data with in the EnKF. This is at least true for perfect model OSSEs. In our OSSE framework for a tornadic supercell storm, apart from the updating of hydrometeor state variables, the updating of water vapor mixing ratio $q_v$ has the largest impact on the overall analysis accuracy followed by vertical wind $w$. However, no updating $w$ leads to larger errors in the flows in the updraft region, including updraft itself and vertical vorticity associated with it. The errors in $q_v$ affect that storm dynamics more indirectly through moist processes.

4. Conclusions and discussions

In this study, the impact of assimilating differential reflectivity $Z_{DR}$ data within an EnKF framework is investigated using observing system simulation experiments with simulated data for a tornadic supercell storm. The Milbrandt and Yau (2005) double-moment microphysics scheme is used in both truth simulation and for EnKF DA; with this double-moment scheme, previous studies have shown the reasonable ability to simulate most important polarimetric radar signatures found in supercell storms. Radar observations are simulated using a polarimetric radar data simulator developed by Jung (2010b), in which T-matrix method is used to calculate the hydrometeor scattering magnitudes for particles of particular sizes. Observation errors of realistic magnitudes are added to the simulated observations, and the same error variances are specified in the EnKF DA. The observation operators from the simulator are also used in the EnKF DA, which is run over a 90 minute period assimilating radar data every 5 minutes spanning the developing and mature stages of the supercell. Experiments are conducted with and without assimilating $Z_{DR}$ data in addition to reflectivity at horizontal polarization $Z_H$ and radial velocity $V_r$ to examine the impact of $Z_{DR}$ assimilation. Results show that the assimilation of $Z_{DR}$ reduces the RMSEs for almost all model state variables at almost all analysis times. The polarimetric signatures of tornadic storm including the $Z_{DR}$ and $K_{DP}$ columns and $Z_{DR}$ arc are all improved. Analyses show that the structures and intensities of hydrometeor fields at both lower and upper levels are improved, even though the strongest $Z_{DR}$ signatures are mostly found at the lower levels due to the concentration of large raindrops there.

Additional sensitivity experiments are conducted to understand the benefit and impact of updating different state variables when assimilating $Z_{DR}$. The first sensitivity experiment excludes the updating of the total number concentrations of all hydrometeors, which are from the use of a double-moment microphysics scheme. Although the number concentrations have very large dynamic ranges, and their relations with $Z_{DR}$ are highly nonlinear, and the ensemble-derived error correlations with $Z_{DR}$ may or may not be reliable enough to produce improved analyses, the results show that updating number concentrations together with the mixing ratios are very beneficial. The number concentrations have high correlations with $Z_{DR}$ at the lower levels that are comparable to those of mixing ratios. If the number concentrations are not updated by $Z_{DR}$ observations, most of
the benefit of assimilating $Z_{DR}$ data is lost, and in fact, for vertical velocity, rainwater and hail mixing ratios, the analysis RMSEs are larger in intermediate DA cycles than those in the experiment not assimilating $Z_{DR}$ data at all. Clearly, updating both mixing ratios and total number concentrations of hydrometeors leads to much more physically consistent analyses.

In other sensitivity experiments, the updating of horizontal wind components, vertical velocity, water vapor or potential temperature by $Z_{DR}$ data is excluded, respectively. This allows us to examine the impact and importance of updating these state variables, which are not directly or are only weakly linked to $Z_{DR}$ via the observation operators. Among these state variables, the updating of water vapor mixing ratio $q_v$ has the largest impact, which is followed by the updating of vertical wind $w$. The updating of horizontal wind components or potential temperature has much smaller though still noticeable impact. Further analysis shows that the updating of $q_v$ or $w$ has significant effects on the intensity and structures of vertical vorticity and vertical velocity in the main updraft region, and significant under-estimation and over-estimation are seen, respectively, in the vertical cross section through the main updraft when $q_v$ or $w$ is not updated. Clearly the updating of $w$ has more direct effect on the storm intensity than the updating of $q_v$, but the effect of the latter via moist processes is apparently very significant. Overall, updating all model state variables when assimilating $Z_{DR}$ data produces the best results, and the RMSEs of analyzed state variables are consistently lower than those of experiment without assimilating $Z_{DR}$ data.

Finally, we point out that the results presented in this paper are limited to OSSE tests with a single supercell storm, and no model error is included. When model error is present, as is with all real data cases, the conclusions may be somewhat different. In addition, other polarimetric measurements, including specific differential phase $K_{DP}$ and co-polar correlation coefficient $\rho_{hv}$, also contain valuable information on the hydrometeors and their PSDs. The assimilation of these parameters and their impact on analyzed storm and subsequent forecasts were not considered in this study, or in the real data study of Putnam et al. (2019); they require further research and investigations. The impact of PRD assimilation on other types of precipitation systems also requires study.

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References


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